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Atmospheric Electricity Potential Gradient at Kew Observatory, 1898 to 1912

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V. *Atmospheric Electricity Potential Gradient at Kew Observatory,* 1898 *to* 1912.

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Contents.

§ 1. In a previous paper,* called E_1 for brevity, I discussed the results obtained for the diurnal variation of the potential gradient of atmospheric electricity at Kew Observatory from 1898 to 1904. The present paper deals with the same subject, but employs data from the fifteen years 1898 to 1912. The earlier period of seven years, though longer than that available at most observatories, was too short to give a satisfactorily representative presentation of some of the phenomena. To obtain results fairly characteristic of the locality many years data are required of some of the meteorological elements, especially barometric pressure and rainfall. For the latter element, in fact, a considerably longer period is desirable than that available even now for potential gradient at Kew. The same may be true of potential gradient itself, but various reasons exist for not waiting longer. Owing to building operations, the electrograph results for 1913 were exposed to special uncertainties. Also the transfer of the electrograph from the position it has occupied since 1898 is now in contemplation. Thus 1912 may be regarded as ending an epoch.

* 'Phil. Trans.,' A, vol. 206, p. 299.

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Another reason requires fuller explanation. The Kew water-dropper—the earliest it is believed in regular operation—was erected in 1861 under Lord KELVIN'S personal supervision. The original electrometer and batteries as they decayed were replaced by others, but the instrument remained essentially unchanged in its original site until 1896. Of the records obtained prior to that date those of only three years had been discussed, two years, 1862 to 1864, by Prof. J. D. EVERETT,* and one year, 1880, by Mr. G. M WHIPPLE.† In both cases the results were expressed in what were really arbitrary units. The relation between the voltage shown by the instrument and the true potential gradient in the open was altogether unknown.

In 1896, after a few years' experience, I recognised the expediency of avoiding a variation in the water pressure which tended to affect the apparent diurnal variation, and of eradicating a shrubbery in the immediate vicinity of the water jet, which presumably influenced the annual variation to a small extent.

§ 2. The importance was also realised of securing that the curve measurements should have a definite meaning. The discharge tube, from freezing of the water or natural decay, required occasional repairs, and had at intervals to be replaced, so that sensible variation in the position of the jet was at least a possibility. To provide for this, regular observations were introduced with a Kelvin portable electrometer in the Observatory garden, at some distance from the building. At first the electrometer was simply placed on the top of a convenient stone pillar. Presently it was realised that the stone pillar, whose diameter is considerable, largely reduced the potential in its immediate neighbourhood, so that the electrometer had to be placed exactly on the same spot if the results obtained on different days were to be comparable. Accordingly a special stand was designed. A rod sliding inside a vertical tube, fixed in the ground, carried at its upper end a small platform. This bore three short vertical pieces at equal intervals round its perimeter, forming a stand just large enough to take the portable electrometer. The sliding rod was designed to admit of observations being taken at two different heights. It was supposed that the reduction of the field due to the presence of the stand, electrometer and observer would be roughly the same at the two heights, so that a fair approach to the true potential gradient in the open would be obtained from the difference of the potentials observed. To reduce the disturbing effect of the observer's presence, a device was introduced allowing manipulation of the electrometer from a greater distance than previously. In practice, however, this device proved troublesome to work. Also the difference—$\frac{1}{4}$ metre—in the two levels at which the potential was observed proved too small, in view of the variability of the potential and the insensitiveness of the electrometer. Some experiments were supposed to show that the disturbing effect due to the apparatus and observer was less than I had anticipated. The outcome was that the special device was laid aside, and observations were confined to one fixed height, about 1·465 metres above ground level.

* 'Phil. Trans.,' vol. 158, 1868, p. 347.

† 'British Association Report for 1881,' p. 443.

Time was not available for elaborate experiments, and suspicions were considerably allayed by the fact that the values obtained for the potential gradient were fully higher than the average of those obtained elsewhere.

§ 3. It was always hoped that an opportunity would present itself for a fuller investigation, but this did not arise until 1909, when Mr. J. S. DINES was attached for a time to the Observatory as student assistant. A number of experiments were made by Mr. DINES. A horizontal bamboo rod, carried in a groove made in a paraffin block, was supported on a small platform, attached to the top of a vertical rod. The vertical rod could slide up and down inside a hollow tube, supported by three tripod legs. The height of the bamboo could be altered by sliding the vertical rod or by altering the stretch of the tripod legs, and the distance to which it projected from the centre of the tripod could be altered by moving it in the groove of the paraffin block. A fine horizontal brass tube fixed to the thinner end of the bamboo held the fuse, and a fine wire passed from the brass tube to a terminal on the thicker end of the bamboo, from which another fine wire led to the portable electrometer, which was supported some yards away on a stand of its own. A second essentially identical apparatus was constructed, and simultaneous observations were made with the fuses of the two at the same level, but projecting to different distances from their respective paraffin blocks. Experiments were also made on the distance the one apparatus had to be from the other to be without sensible effect on it. In another set of experiments the fuse holder was carried by a horizontal wire stretched between insulated supports, borne on vertical poles, a considerable distance apart—a method employed by Mr. C. T. R. WILSON. The potentials obtained in this way and those obtained immediately before and after at the same spot with the bamboo apparatus, when the bamboo projected to the extent finally approved, agreed so closely that one could not say which was the higher. As between the two methods, it thus appeared wholly a question of greater or less convenience. It was found desirable that smoke from the fuse should be blown quite clear of all the apparatus. This was easily secured with the bamboo apparatus, as the mounting enabled it to be readily swivelled round. It might also have been secured with the other apparatus by having two wires stretched in rectangular directions, and using one or the other according to the wind direction. This promised, however, to be somewhat of a complication, accordingly the principle embodied in the bamboo apparatus was adopted.

The new apparatus which was then constructed, and which is still in use, resembles the experimental apparatus in having a vertical rod sliding in a tube, but the latter is sunk to some depth and rigidly fixed in the ground. The sliding rod has towards its upper end an enclosing short hollow tube, which projects to an extent depending on the adjustment of a clamp. This outer tube carries a small platform on which is fixed a paraffin block, serving at once to support and insulate a horizontal bamboo rod. The bamboo has the fuse holder at one end, and at the other a terminal connected by fine wires with the fuse holder and with a portable electrometer at some distance. A shoulder on the long vertical rod, when resting on the top of the ground tube, brings the platform and bamboo rod to the lowest height they can assume. The platform and bamboo can be raised in a few seconds to 1 metre above their original height, by raising the long vertical rod, slipping a pin through a hole in it, and allowing this pin to rest on the top of the ground tube. We thus secure two positions of the fuse, differing in height by exactly 1 metre. To secure that the lower position is exactly 1 metre above ground level, use is made of a vertical rod carried by a flat horizontal board. A permanent mark on this rod is 1 metre above the bottom of the board. The board is placed on the ground, and the position of the clamp on the short vertical tube altered until the axis of the fuse, which projects horizontally, comes exactly level with the mark on the rod. The rod and board are then removed and the observation proceeds. The apparatus was designed by Mr. DINES and myself, the material being supplied and fitted by a local builder. The apparatus, though not a finished workshop article, seems to have hitherto served its purpose reasonably well. A recent remeasurement made the 1- and 2-metre intervals each correct to within 0·25 mm.

In actual use larger uncertainties exist in the height, at least in the lower position. The observations are made over a level piece of turf. But a grass surface, even when newly rolled, is not a mathematical plane, and though the grass is kept short some uncertainty necessarily prevails as to the exact level to which zero potential should be assigned. From the mathematical point of view, it would be better to replace the grass by wood pavement or a carefully levelled flag area. But it is at least open to doubt whether on a warm, still day an artificial surface would be as satisfactory as grass.

§ 4. Absolute observations are made on dry days, usually between 10 and 10.30 a.m. The number of monthly observations is usually from ten to twenty. Observations taken at times when the action of the electrograph appears faulty are left out of account. The results, when apparently satisfactory, are entered in three columns. One contains the potentials observed at the 1-metre level, a second the excess of potential at the 2-metre level over that at the 1-metre level, and the third the potential as measured on the water-dropper curve. This last is deduced from the length of the curve ordinate and the scale value, the latter being determined from time to time by means of the same portable electrometer that is used for the field observations.

Calling the three potential data for the same occasion P_1, P_2, and P_3, the value is found for each month of the ratios $\Sigma P_1/\Sigma P$ and $\Sigma P_2/\Sigma P$. Representing these by r_1 and r_2, $(r_1+r_2)/2$ is accepted as the quantity by which all voltages as measured in the electrograms of that particular month are to be multiplied in order to obtain the corresponding potential gradients in the open.

It seems customary to assume that the factor for converting curve values into true potential gradient is a constant. It is usually determined once for all from a number of observations made under favourable conditions, and not redetermined until some known change is made in the apparatus. Whether this is satisfactory or not depends on the special circumstances of each installation. It would not be wholly satisfactory at Kew, where the discharge tube is long and may develop a sag or have to be replaced. An incidental advantage of regular absolute observations is the check they afford on the working of the apparatus. They help to disclose defects such as poor insulation, and secure more careful attention to the electrograph. One of the reasons for observing potential at 2- as well as 1-metre height, was the fact that the discharge tube of the water-dropper is fully 3 metres above ground level. There is little experimental information as to the differences that may exist in the potential gradient at different small heights above ground level, and there is no general agreement as to the height interval from which the standard potential gradient should be obtained.

§ 5. If V denotes the potential, and z be measured vertically upwards, the potential gradient (dV/dz) must satisfy the two equations

$$d^2V/dz^2 + 4\pi\rho = 0, \quad (1)$$

and

$$(dV/dz)_{z=0} + 4\pi\sigma = 0, \quad (2)$$

where ρ is the volume density, and σ the surface density of the earth's charge.

Unless ρ is zero the potential gradient will vary with the height, and if ρ be known the extent of that variation can be calculated. Various forms of apparatus exist which profess to measure ρ, and one of these—the Ebert apparatus—has been in operation at Kew, with interruptions, since May, 1911. There are, however, reasons for accepting the results obtained with some reserve. It is now generally recognised that the Ebert apparatus takes no account, or only very slight account, of the slow moving heavy ions discovered by LANGEVIN, though it seems to catch the light mobile ions on the whole satisfactorily. MCCLELLAND has found large numbers of Langevin ions near Dublin, and Kew, from its proximity to London, may also not unlikely have large numbers of them, at least with easterly winds. Thus the results derived from the Ebert apparatus as to ρ will be incorrect unless the positive and negative heavy ions are at least approximately equal in number.

In spite of this uncertainty, it seems worth while calculating how the potential gradient may be expected to vary with height at Kew, assuming no free charge in the atmosphere except that resulting from the excess of the positive or negative mobile ions caught by the Ebert apparatus. It will have to be assumed that the distribution of ions is sensibly uniform within 2 metres of the ground because in general the ions observed at Kew are from air at a fixed height, about 2 metres above the ground. The only direct evidence in favour of the hypothesis consists of some experiments made by Mr. GORDON DOBSON* which showed no certain difference in ionic contents

* 'Proceedings Physical Society of London,' vol. 26, 1914, p. 334.

between air collected at the usual level and air taken from immediately above the ground.

The Ebert apparatus determines the charges per cubic centimetre of the free positive ions and the free negative ions separately. The difference between these gives ρ. The results obtained at Kew in 1911 and 1912 have been published monthly in the 'Geophysical Journal' of the Meteorological Office. The 1911 tables give the number of ions as calculated from Sir J. J. THOMSON'S original value of the ionic charge, viz., $3{\cdot}4 \times 10^{-10}$ electrostatic unit. The 1912 tables give the charges per cubic centimetre in electro-magnetic units. For the purpose of calculation it is simplest to employ electrostatic units, *i.e.*, to multiply the numbers in the 1911 tables by $3{\cdot}4 \times 10^{-10}$, and the charges in the 1912 tables by 3×10^{10}. I have similarly dealt with all the available data down to the end of July, 1914. There were several gaps, so that only 25 months' observations were available. Allowing equal weight to the months of the several years, the values obtained for $\rho \times 10^9$ for the 12 months, January to December, were in order

31, 54, 30, 65, 70, 54, 60, 65, 65, 48, 39, 39,

all being plus.

The arithmetic mean of the 12 monthly values gives

$$\rho = +52 \times 10^{-9}.$$

To utilise this result it is perhaps simplest to regard (1) as equivalent to

$$\iint \frac{dV}{dn} dS + 4\pi M = 0, \quad \ldots \ldots \ldots \quad (3)$$

where M is the free charge enclosed by the surface S, of which n denotes the normal. Apply this to a tube of force bounded by 1 sq. cm. of the earth's surface, with the other end on a plane area 1 metre above the ground. Obviously the upper section will be sufficiently approximately 1 sq. cm. We get

$$\begin{aligned} (dV/dz)_{z=0} - (dV/dz)_{z=1\,\text{metre}} &= 4\pi \times 52 \times 10^{-9} \times 10^2 \\ &= 0{\cdot}65_5 \times 10^{-4} \text{ E.U.} \\ &= 0{\cdot}65_5 \times 300 \times 10^{-4} \text{ volts per c.m.} \\ &= 2{\cdot}0 \text{ volts per metre very approximately.} \end{aligned}$$

This signifies a decline of 2 volts per metre in the potential gradient, or practically that the potential gradient deduced from the first metre in the Kew observations should exceed by 2 volts that derived from the second metre.

As will appear presently, the average value of the potential gradient on rainless days at Kew is about 300 volts per metre. Thus we should have as an average, employing r_2 and r_1 as in § 4,

$$r_2/r_1 = 0{\cdot}99_3.$$

Data for r_2/r_1 were available for 48 individual months. The value was 1·00 in 4 cases; in 32 cases it was less, and in 12 cases it was greater than unity. Combining the months of the same name from different years, the means for the 12 months January to December in order were

1·01, 0·96, 0·95, 1·02, 1·01, 0·99, 1·00, 1·00, 0·99, 1·02, 0·95, 0·91,

giving as arithmetic mean $0·98_4$.

The observed departure of r_2/r_1 from unity is in the direction indicated by theory and is of the right order, but is in excess of the calculated difference. A close agreement could hardly be expected on account of the experimental difficulties, and there is the further important fact that the potential gradient observations refer to the forenoon a little after 10 a.m., while the ionic observations refer to about 3 p.m., and the diurnal variation in the ionic charges has not been ascertained.

On examining details, however, it will be recognised that some other factor probably comes in. While the mean value of ρ for the whole 12 months is $+52 \times 10^{-9}$, its mean value for the 6 summer months, April to September, is $+63 \times 10^{-9}$, and that for the 6 winter months only $+40 \times 10^{-9}$.

Thus the decline per metre in the potential gradients in summer and winter should be nearly in the ratio of 3 : 2. The absolute potentials in these seasons, however, as will be seen presently, stand to one another roughly in the ratio of 2 : 3. Thus the departure of r_2/r_1 from unity should on the average be fully twice as large in summer as in winter.

The monthly means recorded above, however, give 1·00 for the summer value of r_2/r_1, as compared with 0·97 for winter. The results for r_2/r_1 during the first year of observation, 1910, fluctuated rather markedly, values in excess of unity being especially frequent in the first few months, when experience was at a minimum. If we omit all data for 1910, the summer value of r_2/r_1 falls to 0·99, the winter value remaining 0·97, and the mean for the whole year becomes $0·98_0$.

As before, the summer value of r_2/r_1 exceeds the winter one. Assuming the difference to be real, a probable explanation immediately suggests itself. The extent of level turf in the Observatory garden is limited. It is immediately surrounded by ground devoted to vegetables, and this in turn is enclosed by a hedge about $4\frac{1}{2}$ feet high. The vegetation must inevitably have some slight effect in lowering the potential at small heights, and this effect will naturally be greatest at the season when vegetation is most exuberant. Also while the grass at the place of observation is mown more frequently in summer than in winter, its average height is probably greater at the former season. These disturbing influences will naturally be felt mainly, if not entirely, by the potential observations at the 1-metre level.

§ 6. Before concluding his engagement at the Observatory, Mr. DINES took some observations intended to determine the relation between the potential gradients obtained in the old way with the old apparatus, and those obtained in the new way.

This was less easy than might appear at first sight. The ground immediately round the old stand was gravel, and slightly different in level from the surrounding grass. The observations with the old apparatus referred to a point directly over the stand, which was a fixture. There was thus no possibility of taking observations with the old and the new apparatus at exactly the same spot. The observations, moreover, were not so numerous as might have been desired. The mean result made the potential gradient deduced by the old and the new apparatus stand to one another in the ratio 1·00 to 1·65.

When the comparison was made there were several fruit trees near the grass plot, and the observations made with the new apparatus suggested the expediency of their removal. It was accordingly decided to cut down the trees at the year's end, and to start the new year, 1910, with the new apparatus. Unfortunately circumstances did not allow of any direct determination of the effect of removing the trees. Thus Mr. DINES' observations did not suffice for the determination of a factor to be applied to results obtained prior to 1910 to bring them to what they would have been under the conditions since prevailing. This, however, can, I think, be done fairly satisfactorily by considering the values obtained for the mean annual potential gradient. These have been as follows :—

	Old apparatus.												New apparatus.		
Year . . .	1898.	1899.	1900.	1901.	1902.	1903.	1904.	1905.	1906.	1907.	1908.	1909.	1910.	1911.	1912.
P.G. . . .	161	179	141	156	145	162	167	167	156	163	148	164	310	301	300

If we divide the 12 years during which the old apparatus was in use into the period 1898 to 1904, treated in E_1, and the later period 1905 to 1909, we find 158·7 as the mean potential for the former period, and 159·6 as the mean potential for the latter. There is thus no indication of any progressive change in the value of the potential gradient. Moreover, the mean derived from any three successive years departs but little from the mean 159·1 derived from the whole 12. We are thus unlikely to be much in error if we regard the means 159·1 and 303·7, derived respectively from the years when the old and the new apparatus was in use, as representing the same real potential gradient. This gives 1·91 as the factor to be applied to results obtained prior to 1910 to bring them up to what they would have been if the new apparatus had been in use under the conditions now existing.

When preparing E_1 I did not suspect that so large an underestimate was being made of the absolute value of the potential gradient, but I quite realised that a sensible correction was probably necessary, and for that reason, amongst others, a number of the results were expressed as ratios, not as absolute potentials. Even now it cannot be claimed that we are getting an absolutely full measure of the potential gradient in the open. The influence of the apparatus and observer it is believed is small, but it may not be wholly infinitesimal. Then the site—apart from the peculiarities already mentioned—does not altogether adequately represent an infinite plane. There are now no fruit trees within 65 feet of the ground tube, but there are several low buildings in the neighbourhood, one coming within about 70 feet of the site. Finally the main Observatory building, the elevation of whose dome above the garden ground is fully 60 feet, is only about 60 yards away, and there are four elm trees at distances of from 55 to 80 yards, still of considerable height though they lost their tops many years ago. There are sites less open to criticism in the Old Deer Park outside the Observatory enclosure, but the time required to visit a distant site would be a serious obstacle to its regular use, even if it were available.

§7. In computing the following tables all potential data for years prior to 1910 have been multiplied by 1·91. The spot to which all absolute values refer must be regarded as the site of the apparatus for absolute observations, which is about 20 feet above mean sea level in lat. 51° 28′ N, long. 0° 19′ W.

Table I. gives the mean value of the potential gradient for each of the 180 months of observation, with corresponding means for each year and each of the 12 months. The results are from selected rainless days, free from negative potential, 10 in each month, except in a few cases where 8 or 9 days only were available.

TABLE I.—Mean Monthly and Yearly Values of Potential Gradient (volts per metre).

Year.	January.	February.	March.	April.	May.	June.	July.	August.	September.	October.	November.	December.	Mean.
1898	332	466	413	208	185	199	199	239	311	308	477	351	308
1899	544	569	388	265	288	201	172	243	159	334	319	626	342
1900	361	348	344	204	183	139	164	199	225	254	300	502	269
1901	455	472	269	267	243	218	181	166	223	365	411	309	298
1902	313	443	290	285	178	220	164	212	199	235	340	453	277
1903	309	332	365	315	319	298	201	246	258	269	329	477	309
1904	371	363	338	306	248	202	223	225	248	281	495	521	319
1905	418	323	315	311	248	235	258	290	319	309	462	340	319
1906	363	380	351	300	180	241	241	202	273	267	321	466	298
1907	378	435	367	281	275	191	296	220	237	239	432	372	311
1908	527	304	399	357	197	210	168	199	199	216	336	288	283
1909	382	397	309	380	294	166	168	254	248	235	432	477	313
1910	399	432	386	262	273	203	232	202	206	330	450	350	310
1911	457	345	378	286	249	219	209	217	250	373	366	269	301
1912	523	337	228	271	204	167	259	231	334	399	316	332	300
Mean .	409	396	343	287	238	207	209	223	246	294	386	409	304

Even fair weather days in the same month vary immensely amongst themselves electrically, and there is a good deal that is "accidental" in means derived from 10 days only. When the rainless days available are largely in excess of 10, as is frequently the case, the natural tendency, at least in summer, is to prefer days in which the potential is high to those in which it is low. The former usually show the diurnal variation more clearly, and there is in their case a greater presumption against bad insulation, a not infrequent occurrence in damp weather, or at seasons when spiders are most busy. Thus the mean potential in a month when the choice of days is small is apt to come low. There are, however, individual winter months in which the contrary tendency prevails. During fog the potential at Kew is often much above the average. If only part of a day is foggy one prefers to omit it, because the diurnal variation shown is largely dependent on the accident of what hours the fog was incident. If, however, the choice of days is very restricted, one is obliged to include days of intermittent fog. Again the number of days available depends considerably on how the apparatus has been working. At times there are somewhat numerous defects, whether from poor insulation, freezing of the jet, accident or lack of attention, which restrict the choice, leading to the same consequences as frequent rain. Finally sensible changes sometimes take place between two successive scale determinations, leading to uncertainty in the scale values to be applied. Thus "accident" plays some part in the individual monthly values in Table I.

Judging by the fluctuations in the values for months of the same name, a longer period would be desirable. The smoothness in the annual variation derived from the 15 years is, however, truly remarkable in view of the irregularities visible in the corresponding variations deduced from either of the sub-periods 1898 to 1904 or 1905 to 1912. It can hardly be doubted that it is a fair approximation to normal conditions.

The results are shown graphically in the top curve of fig. 4, p. 142, unity representing 304 volts, the mean value for the whole year. The maximum and minimum appear respectively at about midwinter and midsummer, and so somewhat in advance of the times of minimum and maximum of temperature at ground level. They are naturally still more in advance of the times of minimum and maximum temperature at any depth underground. This suggests that direct solar radiation has more to do with the annual variation than earth temperature or meteorological conditions in the lower strata of the atmosphere.

The extent of the annual variation in potential gradient is pretty similar to that in vapour pressure or density. But the minima of these elements at Kew occur in February and the maxima in July, while June and September values are closely alike. Thus a formula of the type once proposed by ELSTER and GEITEL $dV/dn = A/(1+kq_0)$, where A and k are positive constants and q_0 vapour density, cannot be made to fit the Kew data very satisfactorily.

§ 8. In E_1 conspicuous differences were pointed out between the diurnal inequalities derived from 1898 to 1904 and those derived from earlier epochs. As this might arise either from the periods available having been too short to eliminate accidental features or form a real change in the nature of the phenomena, data are given here for the period 1905 to 1912 as well as for the whole 15 years. Between 1901·5 and 1909·0, which may be regarded as the epochs to which refer the data in E_1, and those from the years 1905 to 1912, there was considerable growth in Western London, and if the difference between EVERETT's results for 1862–4 and mine for 1898 to 1904 were due to urban extension, then the data for 1905 to 1912 would naturally depart still further from EVERETT's. Table II. gives the diurnal inequalities for the whole 15 years, and Table III. those for the seven years 1905 to 1912. The highest and lowest hourly values are in heavy type. The hours are really G.M.T., but local time is only $1\frac{1}{4}$ minutes after Greenwich. For purposes of comparison data from Table III., p. 306, of E_1 must be multiplied by 1·91.

Besides the diurnal inequalities for the 12 months, Tables II. and III. contain diurnal inequalities for the whole year and for three seasons, winter (November to February), equinox (March, April, September and October), and summer (May to August). The inequalities for the 12 months in Table II. are shown graphically in fig. 1, p. 146. This is immediately comparable with fig. 2 of E_1, provided the line there marked 50 volts be regarded as representing 95. To facilitate recognition of the resemblances and differences between the data from different epochs, fig. 2, p. 147, shows in juxtaposition the diurnal inequalities for the year and the three seasons derived from 1898 to 1904 (upper curves), the whole 15 years (central curves), and 1905 to 1912 (lower curves); fig. 3, p. 148 contains isopleths for the diurnal variation of the 15 years. For it I am indebted to Mr. E. H. NICHOLS, B.Sc., professional assistant at Kew Observatory. To assist the eye, the isopleth curves are drawn thicker or thinner according as they answer to potentials which are greater or less than the mean for the year, 304 volts per metre. The two broken lines indicate respectively the times of sunrise and sunset throughout the year.

A careful comparison of Tables II. and III., or of corresponding curves in fig. 1 and in fig. 2 of E_1, shows differences between the monthly diurnal inequalities derived from different epochs, but some of these are undoubtedly "accidental." In the seasonal diurnal inequalities in fig. 2, where "accident" is more completely eliminated, the differences that exist—except as regards the relative depths of the morning and afternoon minima—make small appeal to the eye. It requires an auxiliary such as analysis into Fourier series to show their nature distinctly. It may reasonably be inferred that no large rapid change is in progress in electrical conditions at Kew, and that the inequalities derived from the 15 years give a close approach to normal conditions. Whether the results are sensibly dependent on the particular site selected for the water-dropper it is impossible to say until adequate data are available from a different site.

A glance at fig. 1 shows a conspicuous difference between midsummer and midwinter. The time elapsing between the forenoon and afternoon maxima and the depth of the

TABLE II.—Diurnal Inequality

Hour (G.M.T.).	1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.
January . .	−51·9	−69·0	−88·5	−**104·1**	−97·0	−75·3	−31·8	+14·9	+50·4	+61·3	+39·4	+24·0
February . .	−39·0	−64·7	−87·3	− **96·6**	−91·8	−76·4	−32·4	+23·4	+62·3	+**77·1**	+50·7	+19·4
March . . .	−19·8	−46·6	−69·0	− **78·1**	−68·3	−38·9	+ 4·1	+38·8	+58·6	+45·7	− 1·1	−33·3
April . . .	−14·4	−37·7	−52·4	− **59·0**	−46·6	−18·7	+25·2	+54·1	+50·6	+23·0	−17·7	−40·1
May	− 3·4	−21·3	−35·9	− 40·9	−27·2	− 6·9	+19·3	+37·7	+36·0	+16·1	−13·9	−29·7
June . . .	− 9·9	−23·8	−33·8	− 35·2	−20·3	− 3·7	+19·7	+39·2	+38·0	+24·9	+ 1·4	−18·0
July	−22·8	−40·4	−42·6	− **43·6**	−30·6	− 8·2	+29·0	+54·0	+**58·2**	+36·8	+ 3·1	−20·5
August . . .	−25·4	−41·7	−**53·3**	− 49·2	−36·4	− 5·4	+29·3	+52·9	+55·4	+32·4	+ 3·2	−15·8
September . .	−33·6	−49·5	−55·1	− **60·1**	−50·3	−24·0	+ 6·3	+32·9	+36·8	+21·5	− 4·2	−16·9
October . .	−50·5	−57·0	−**57·5**	− 52·5	−38·4	−13·4	+19·0	+54·4	+**61·8**	+43·2	+ 5·5	−18·2
November . .	−43·8	−59·3	−68·1	− **69·3**	−58·0	−36·4	− 6·2	+26·8	+41·3	+44·5	+15·9	− 6·5
December . .	−57·3	−76·0	−84·4	− **88·9**	−82·8	−63·7	−30·1	+15·6	+47·5	+53·7	+29·9	+13·7
Year . . .	−31·0	−48·9	−60·7	− **64·8**	−54·0	−30·9	+ 4·3	+37·1	+49·7	+40·0	+ 9·4	−11·8
Winter . . .	−48·0	−67·3	−82·1	− **89·7**	−82·4	−63·0	−25·1	+20·2	+50·4	+59·2	+34·0	+12·7
Equinox . .	−29·6	−47·7	−58·5	− **62·4**	−50·9	−23·8	+13·6	+45·1	+52·0	+33·4	− 4·4	−27·1
Summer . .	−15·4	−31·8	−41·4	− 42·2	−28·6	− 6·0	+24·3	+46·0	+46·9	+27·6	− 1·6	−21·0

TABLE III.—Diurnal Inequality

Hour (G.M.T.).	1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.
January . .	−63·2	−82·8	−108·0	−**125·8**	−111·3	−84·6	−36·7	+20·1	+59·5	+63·8	+44·3	+30·7
February . .	−40·6	−63·0	− 87·4	− **94·8**	− 90·5	−79·0	−35·5	+21·9	+59·1	+73·9	+48·6	+13·8
March . . .	−33·9	−62·5	− 84·6	− **93·9**	− 83·9	−51·4	− 1·0	+38·6	+62·4	+57·0	+13·4	−20·2
April . . .	−20·6	−51·4	− 70·0	− **80·4**	− 68·1	−43·1	+ 1·9	+36·0	+43·4	+23·0	−11·8	−30·7
May	−17·0	−28·5	− 52·6	− **56·8**	− 40·0	−17·4	+14·5	+37·4	+43·5	+26·8	+ 2·1	−12·5
June . . .	−12·8	−26·5	− 38·4	− **41·8**	− 26·5	− 9·9	+15·1	+42·2	+40·6	+25·4	+ 3·0	−13·3
July	−27·4	−53·8	− **55·7**	− 52·9	− 40·3	−20·8	+18·7	+52·3	+**63·4**	+42·7	+ 8·9	−19·0
August . . .	−30·8	−46·8	− **55·7**	− 47·1	− 35·6	− 1·4	+38·9	+**64·0**	+62·4	+36·8	+ 0·1	−17·2
September . .	−48·9	−63·4	− 71·0	− **76·9**	− 62·5	−33·9	+ 0·5	+32·9	+37·4	+26·4	+ 4·9	− 7·6
October . .	−54·9	−64·6	− **66·7**	− 59·4	− 41·9	−18·3	+11·6	+49·3	+60·7	+49·2	+10·8	−10·3
November . .	−54·2	−71·4	− 82·8	− **89·0**	− 78·1	−53·7	−13·8	+31·8	+55·5	+**57·0**	+30·6	+ 2·7
December . .	−67·4	−80·0	− 82·4	− **82·6**	− 75·8	−59·2	−35·8	+18·8	+50·2	+50·1	+32·3	+13·0
Year . . .	−39·3	−58·7	− 71·3	− **75·1**	− 62·9	−39·4	− 1·0	+37·1	+53·2	+44·3	+15·6	− 5·9
Winter . . .	−56·4	−74·3	− 90·2	− **98·1**	− 88·9	−69·1	−28·0	+23·1	+56·1	+61·2	+38·9	+15·0
Equinox . .	−39·6	−60·5	− 73·1	− **77·6**	− 64·1	−36·7	+ 3·3	+39·2	+51·0	+38·9	+ 4·3	−17·2
Summer . .	−22·0	−41·4	− **50·6**	− 49·6	− 35·6	−12·4	+21·8	+49·0	+52·5	+32·9	+ 3·5	−15·5

1898 to 1912 (volts per metre).

13.	14.	15.	16.	17.	18.	19.	20.	21.	22.	23.	24.	Range.	A.D.
+ 8·6	+ 1·3	+ 7·2	+21·2	+43·1	+53·9	**+63·5**	+61·0	+49·6	+37·1	+ 9·5	−28·3	167·6	45·5
−14·0	−32·5	−24·9	−11·0	+19·1	+48·4	+71·6	+68·8	+67·8	+51·3	+21·8	−11·4	173·7	48·5
−47·9	−50·5	−44·2	−33·0	− 7·6	+29·3	+71·1	**+85·7**	+80·4	+72·5	+43·5	+ 7·9	163·8	44·8
−53·7	−52·3	−47·0	−38·6	−23·0	+ 9·4	+55·2	+82·9	**+83·6**	+69·5	+39·3	+ 8·5	142·6	41·8
−44·2	**−53·0**	−45·3	−38·9	−26·8	+ 3·2	+40·9	+60·6	**+69·3**	+57·5	+34·2	+12·8	122·3	32·3
−32·9	−40·3	**−41·3**	−35·1	−20·4	+ 2·9	+20·9	+41·8	**+52·1**	+43·2	+23·8	+ 6·5	93·4	26·2
−29·3	−36·9	−36·5	−32·6	−21·4	− 5·1	+18·2	+42·7	+57·2	+48·3	+25·2	− 2·4	101·8	31·1
−26·0	−39·0	−44·4	−38·9	−23·1	+ 3·6	+34·5	+56·6	**+62·1**	+49·4	+26·5	− 7·3	115·4	33·8
−23·2	−25·9	−21·4	−11·2	+12·7	+44·0	+66·4	**+67·5**	+56·4	+37·6	+ 9·6	−15·4	127·6	32·6
−31·3	−33·7	−20·1	− 0·8	+32·9	+54·9	+57·7	+48·2	+31·3	+10·4	−10·7	−35·9	119·3	35·0
−14·4	−10·2	+ 7·3	+25·6	+44·2	**+49·4**	+48·9	+47·3	+38·7	+16·3	− 6·1	−28·6	118·7	33·9
+ 8·7	+ 7·2	+18·8	+37·2	+49·9	**+59·6**	**+59·6**	+50·8	+39·6	+24·1	+ 0·7	−33·5	148·5	43·1
−25·0	−30·6	−24·3	−13·0	+ 6·6	+29·5	+50·7	**+59·5**	+57·3	+43·1	+18·1	−10·6	124·3	33·8
− 2·8	− 8·5	+ 2·1	+18·3	+39·1	+52·8	**+60·9**	+57·0	+48·9	+32·2	+ 6·5	−25·4	150·6	41·2
−39·0	−40·6	−33·2	−20·9	+ 3·8	+34·4	+62·6	**+71·1**	+62·9	+47·5	+20·4	− 8·7	133·5	37·2
−33·1	**−42·3**	−41·9	−36·4	−22·9	+ 1·2	+28·6	+50·4	**+60·2**	+49·6	+27·4	+ 2·4	102·5	30·4

1905 to 1912 (volts per metre).

13.	14.	15.	16.	17.	18.	19.	20.	21.	22.	23.	24.	Range.	A.D.
+ 7·2	+ 8·6	+22·0	+35·4	+54·7	+63·3	+71·2	**+ 74·4**	+57·5	+36·2	+ 2·2	−38·3	200·2	54·7
− 8·2	−29·5	−24·9	−11·4	+19·9	+52·5	**+75·4**	+ 70·4	+71·6	+53·0	+22·4	−16·6	170·2	48·5
−33·4	−35·9	−26·5	−20·8	+ 3·4	+34·9	+73·6	**+ 80·6**	+75·5	+67·4	+39·4	+ 0·6	174·5	45·6
−42·5	−39·8	−31·1	−25·9	− 9·1	+21·2	+70·5	**+102·4**	+99·0	+76·2	+42·3	+ 8·6	182·8	43·7
−29·1	−41·1	−34·8	−32·9	−21·9	+ 9·3	+48·6	+ 65·6	**+70·2**	+55·3	+23·2	− 2·4	127·0	33·1
−25·1	−30·5	−30·6	−28·5	−12·4	+ 9·1	+21·9	+ 40·4	**+49·8**	+34·0	+14·9	− 0·1	91·6	24·7
−24·0	−29·7	−26·8	−26·3	−14·1	+ 3·0	+27·0	+ 47·2	+56·6	+49·9	+26·6	− 5·4	119·1	33·0
−22·6	−39·0	−44·0	−40·4	−25·9	+ 0·3	+34·0	+ 57·1	+62·2	+44·0	+20·0	−13·0	119·7	35·0
−14·8	−13·4	−10·6	− 0·2	+25·4	+49·1	+70·1	**+ 75·8**	+60·8	+37·0	+ 5·1	−23·4	152·7	35·5
−15·2	−13·7	− 0·2	+12·4	+48·8	**+61·0**	+55·2	+ 35·8	+13·8	− 5·4	−19·6	−38·0	127·7	34·0
− 5·7	− 3·2	+16·2	+32·4	+45·2	+51·3	+52·3	+ 52·7	+40·0	+21·2	− 5·6	−30·7	146·0	40·7
+ 6·2	+ 7·8	+23·3	+42·7	+55·6	**+63·8**	+58·8	+ 50·2	+35·9	+15·2	− 8·3	−41·2	146·4	43·6
−17·3	−21·6	−14·0	− 5·3	+14·1	+34·9	+54·9	**+ 62·7**	+57·7	+40·3	+13·6	−16·7	137·8	35·7
− 0·1	− 4·1	+ 9·1	+24·8	+43·8	+57·7	**+64·4**	+ 61·9	+51·2	+31·4	+ 2·7	−31·7	162·5	45·1
−26·5	−25·7	−17·1	− 8·6	+17·1	+41·6	+67·4	**+ 73·7**	+62·3	+43·8	+16·8	−13·0	151·3	38·3
−25·2	−35·1	−34·1	−32·0	−18·6	+ 5·4	+32·9	+ 52·6	**+59·7**	+45·8	+21·2	− 5·2	110·3	31·4

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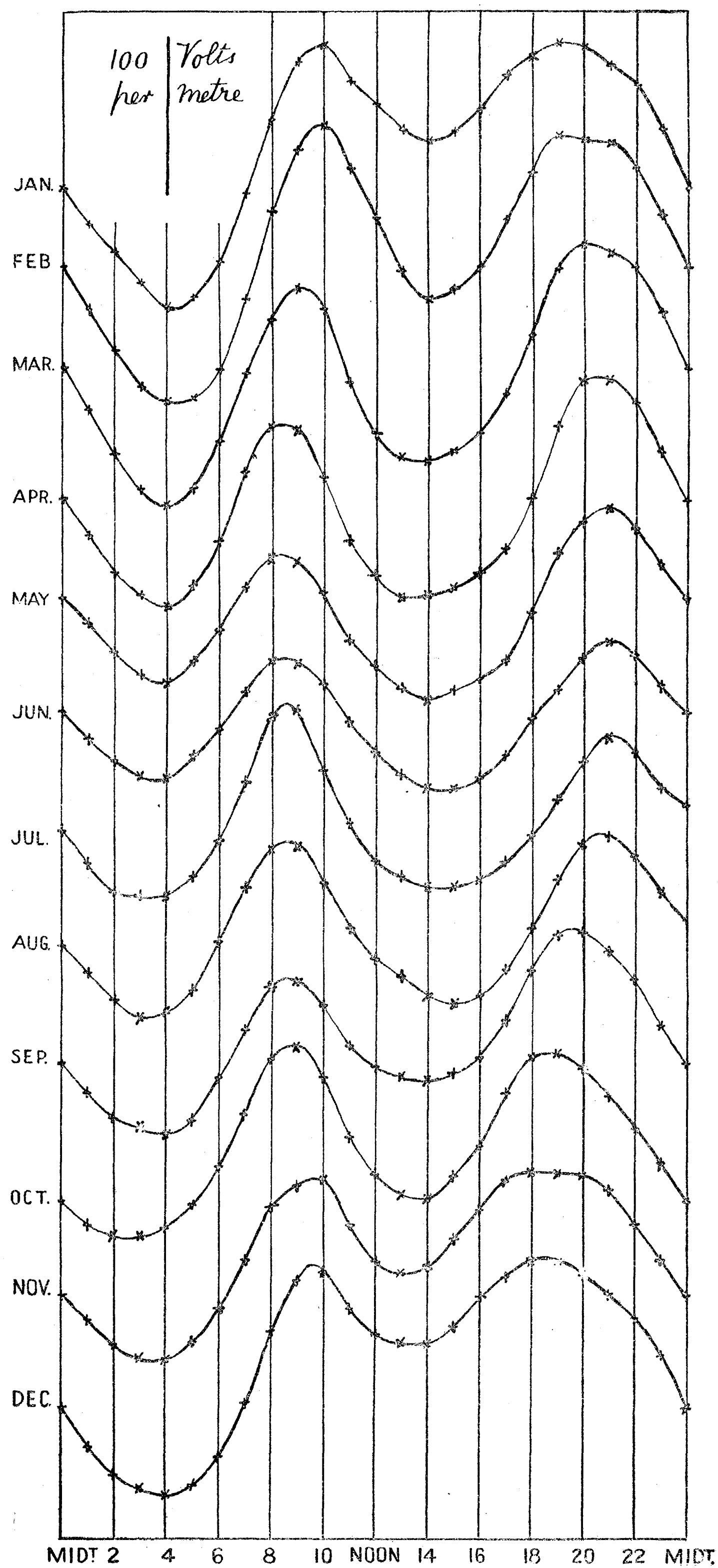

Fig. 1. Diurnal inequalities, 1898–1912.

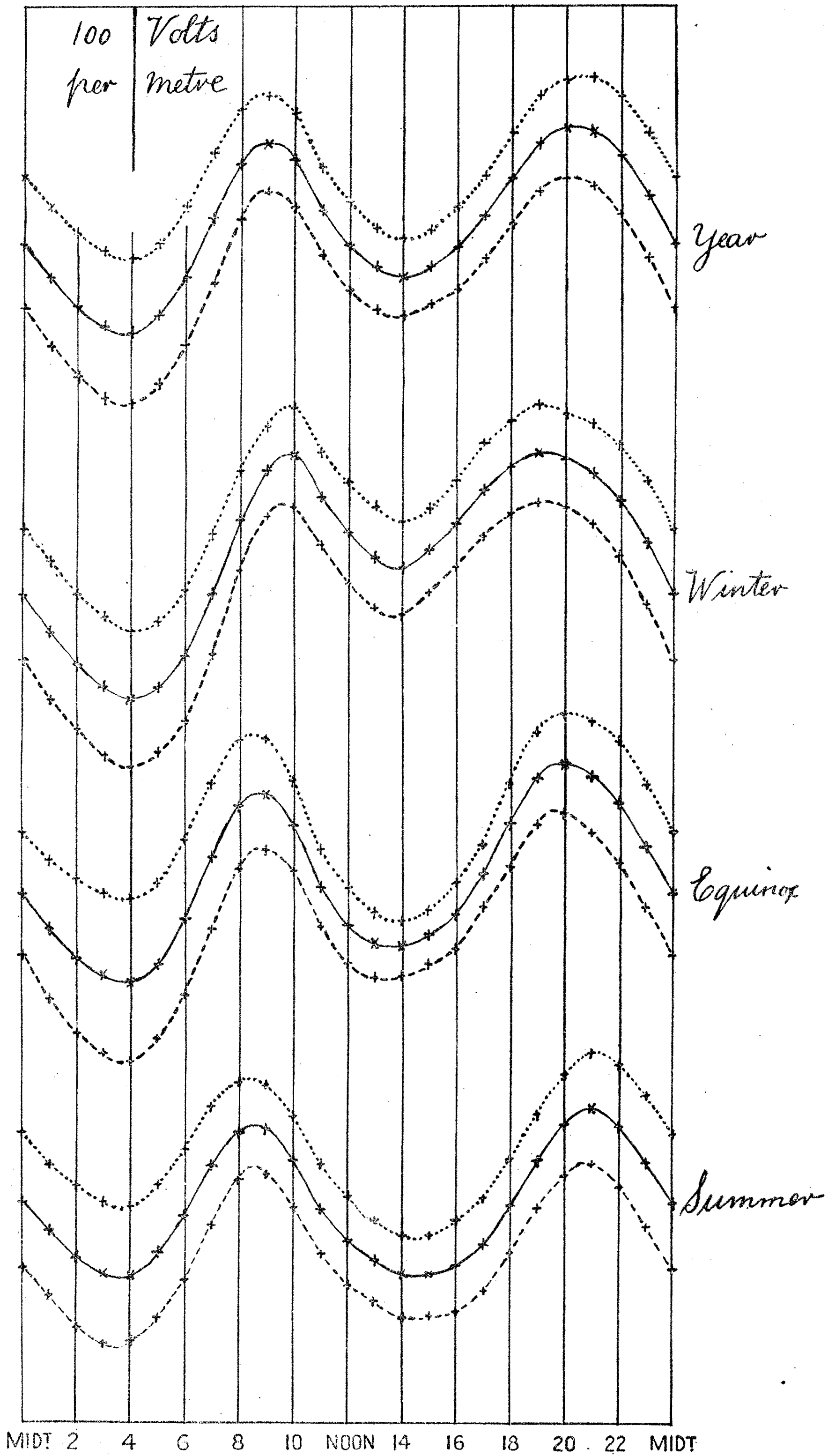

Fig. 2. Diurnal inequalities.

×·····× 1898–1904. ×——× 1898–1912. ×-----× 1905–1912.

intervening minimum diminish as we approach midwinter. But the transition is so gradual that it is difficult to draw a line between summer and winter months. This is one of the reasons for dividing the year into three seasons.

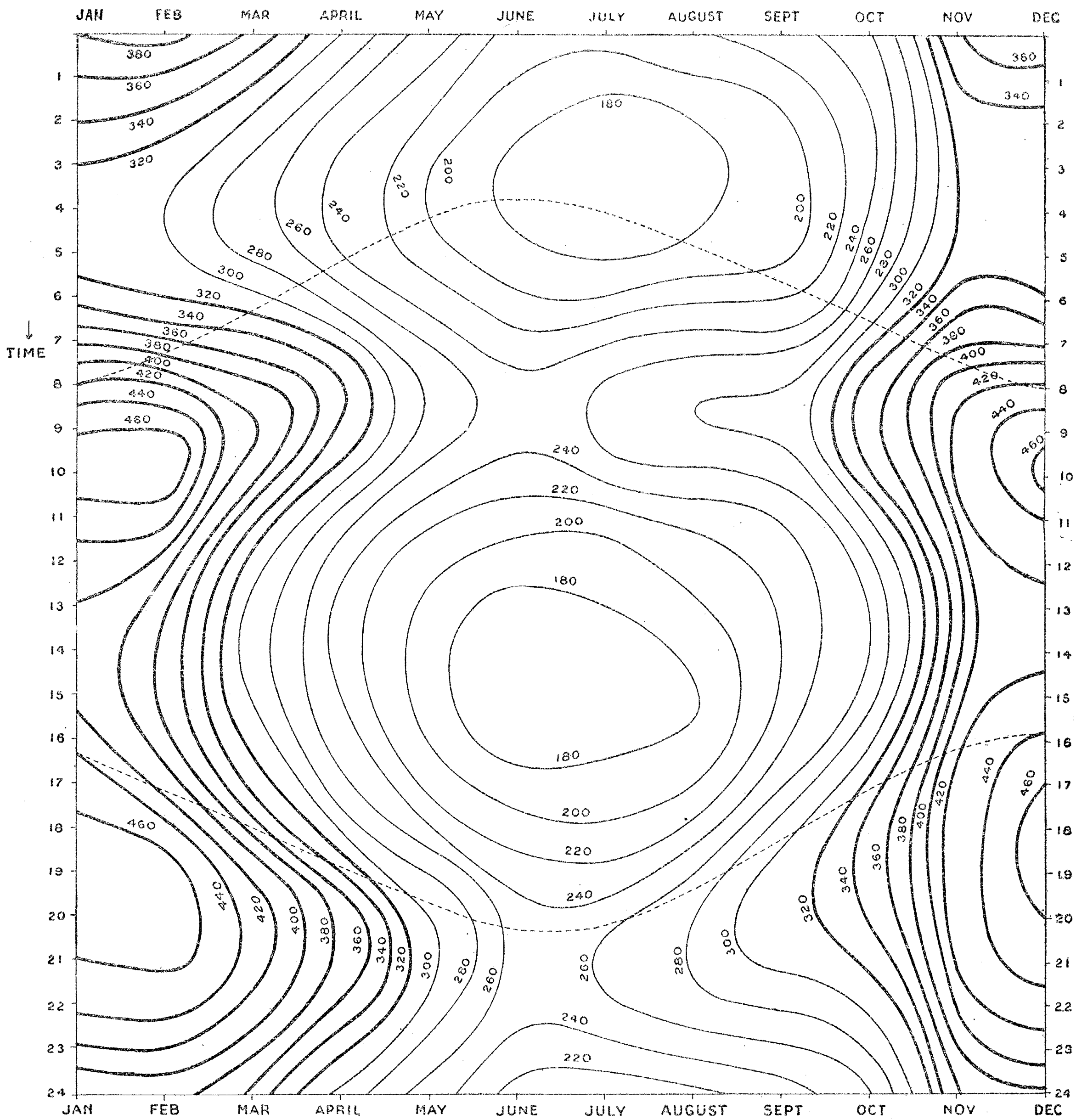

Fig. 3. Isopleths.

▬▬▬ Potentials above mean.
——— ,, below ,,
- - - - - - Times of sunrise and sunset.

The November, December, and January curves are of very similar type. The February curve resembles the March curve almost as closely as it does the January curve, and but for the desirability of having the three seasons equal, February might have been included under equinox and April under summer.

When the type of the diurnal inequality varies but little with the season the range usually gives a very fair idea of how the intensity of the forces to which the regular

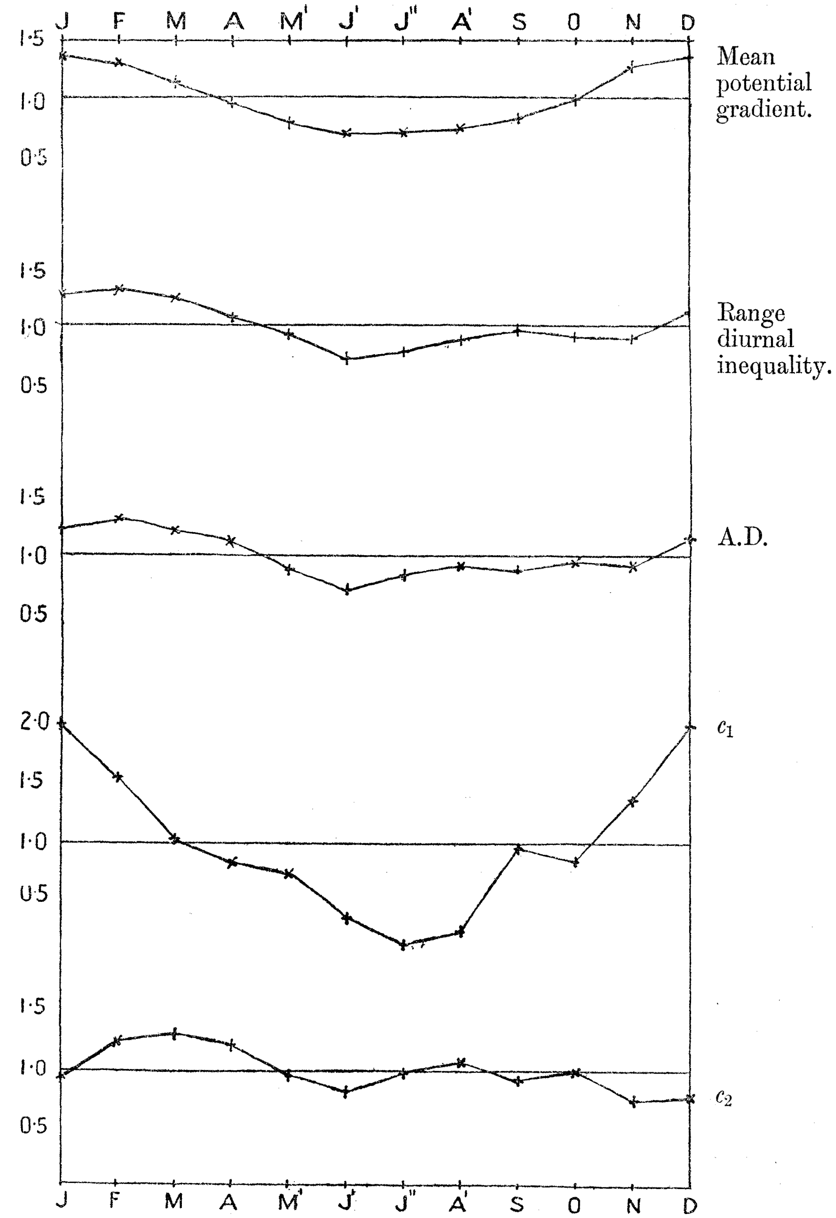

Fig. 4. Annual variation.

diurnal changes are due vary throughout the year. An exception to this may arise when the curves are of a sharply peaked type, unless the times of daily maximum and minimum happen to fall very close to exact hours. Peaked curves are, however, unusual in the diurnal inequalities of natural phenomena.

If the type varies much, a better idea of the activity of the forces is derived from

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the quantity termed A.D. (signifying *average departure* from the mean) in Tables II. and III. This represents the sum of the 24 hourly differences from the mean—taken regardless of sign—divided by 24.

The ranges and average departures expressed in terms of their arithmetic means are represented in the second and third curves of fig. 4. Both curves show a maximum in February and a minimum in June, with at least a suggestion of a secondary maximum in Autumn and a secondary minimum in November. But the data from 1905 to 1912 put the principal maximum in January and give a less clear indication of a secondary maximum and minimum, the latter appearing to fall in October. Thus while there is clearly a principal maximum towards midwinter and a principal minimum towards midsummer, the precise times of their occurrence, and the existence of a secondary maximum and minimum are questions it would require a still longer period of years to settle satisfactorily.

It should be remembered that when data from a large number of years are combined there are two distinct causes which lead to large diurnal inequalities, firstly the persistent occurrence of large amplitudes, and secondly close agreement in the times of maximum and minimum in months of the same name. If the phase varies much from year to year, data from different years may to a considerable extent neutralise one another, and we may form an inadequate idea of the average intensity of the forces causing the diurnal variation.

§ 9. Our ignorance of the cause of the diurnal inequality renders it difficult to say

TABLE IV.

Month or season.	Ratios borne to mean monthly or seasonal values of potential gradient.			$\frac{\text{Night fall}}{\text{Day fall}}$.
	Inequality range.	Day fall.	Night fall.	
January	0·41	0·15	0·41	2·79
February	0·44	0·28	0·42	1·53
March	0·48	0·32	0·48	1·50
April	0·50	0·38	0·50	1·32
May	0·51	0·38	0·46	1·21
June	0·45	0·38	0·42	1·10
July	0·49	0·46	0·48	1·06
August	0·52	0·45	0·52	1·16
September	0·52	0·25	0·52	2·04
October	0·41	0·32	0·39	1·21
November	0·31	0·15	0·31	2·02
December	0·36	0·11	0·36	3·19
Year	0·41	0·26	0·41	1·55
Winter	0·38	0·17	0·38	2·22
Equinox	0·46	0·32	0·46	1·44
Summer	0·47	0·41	0·47	1·15

what features are of most importance. Any departure from the mean, whether rise or fall, is followed sooner or later by a change in the other direction. Which change is the most fundamental, or whether they possess equal significance, are questions to which no reply is at present possible. The fact that the two falls shown by the diurnal inequality occur, the one wholly in night hours the other wholly in day hours, suggest these falls for consideration in preference to the forenoon and afternoon rises of potential. They are compared in Table IV. with one another and with the range of the diurnal inequality. When the principal maximum occurs in the afternoon and the principal minimum in the morning, as is generally the case at Kew, the night fall and the range are identical.

The quantities in the three first columns of Table IV. are expressed as fractions of the corresponding mean monthly values given in Table I. The ratio borne by the inequality range to the mean value for the month is much less variable than is the absolute value of the range, and a similar remark applies to the night fall. On the other hand, the day fall actually diminishes as the monthly mean increases. At midsummer it is very nearly as large as the night fall, but at midwinter it is only about one-third as large as the latter.

§ 10. A positive value in the potential gradient implies a corresponding negative charge on the earth, the surface density being given by (2) of § 5. Any change in the gradient implies a change in the surface density. The change in the charge on a sq. cm. of the earth's surface shows only the difference between what enters and leaves it. The change may represent but a small fraction of the current traversing the element of surface. The increment to the charge may also arise from a vertical current or a horizontal current, or partly from the one and partly from the other. As a matter of fact, we know that usually a vertical air-earth current exists, as well as so-called earth currents whose direction is mainly at least horizontal. Earth currents are much in evidence during magnetic storms, but no certain relation between potential gradient changes and magnetic storms seems yet to have been established. The normal diurnal inequality of potential gradient could be connected only with normal air or earth currents, and information about either of these phenomena is limited. On rainless days the air-earth current seems normally to be directed downwards throughout the whole 24 hours, so it alone cannot account for changes in the surface charge. It is, however, of some interest to compare the relative sizes of the air-earth current and the alterations of surface charge calculated from the changes in potential gradient. February is the month which shows the largest daily changes in Table V. The hourly changes in it going to the nearest volt are as follows:—

Hour	0–1.	1–2.	2–3.	3–4.	4–5.	5–6.	6–7.	7–8.	8–9.	9–10.	10–11.	11–12.
Forenoon . . .	−28	−26	−23	−9	+5	+15	+44	+56	+39	+15	−26	−31
Afternoon . .	−33	−19	+8	+14	+30	+29	+23	−3	−1	−16	−30	−33

When the sign is plus the surface density is increasing numerically. The total increase in the negative charge per sq. cm. from 4 a.m. to 10 a.m. amounts to 153×10^{-15} coulombs, and the most rapid hourly increase—occurring between 7 and 8 a.m.—is 49×10^{-15} coulombs. This is equivalent to a steady current of $1 \cdot 36 \times 10^{-17}$ amperes per sq. cm. The average value of the air-earth current, as measured at various stations, is in the neighbourhood of 2×10^{-16} amperes per sq. cm., or about 15 times as big as the above current. Thus in general the changes in surface density must represent the balance between a much larger air-earth current and earth currents whether vertical or horizontal.

§ 11. Table V. gives the amplitudes and phase angles of the 24-, 12-, 8- and 6-hour Fourier "waves" in the diurnal inequalities derived from the whole 15 years, and

TABLE V.—Diurnal Inequality 1898 to 1912. Amplitudes and Phase Angles.

Month or season.	c_1.	α_1.	c_2.	α_2.	c_3.	α_3.	c_4.	α_4.
		°		°		°		°
January	57·72	209·8	46·23	174·4	14·39	22	5·47	261
February	44·43	201·8	61·41	172·4	18·81	5	5·24	238
March	29·06	156·9	64·50	183·1	10·58	45	8·78	301
April	23·18	128·1	60·02	187·8	7·04	95	12·52	312
May	20·82	108·2	46·86	185·9	3·31	100	7·69	317
June	9·69	97·9	40·52	184·1	3·16	89	4·32	304
July	2·95	118·0	47·97	183·9	7·04	110	8·67	285
August	6·29	151·0	52·63	185·5	3·87	123	7·76	317
September	26·97	183·8	45·18	196·7	5·63	28	7·15	331
October	23·89	222·0	48·50	209·4	14·17	30	3·92	290
November	39·13	211·5	36·60	198·5	13·22	42	5·44	253
December	57·52	212·9	37·81	184·5	13·18	33	6·57	241
Arithmetic means	28·47	—	49·02	—	9·53	—	6·96	—
Year	23·16	190·4	48·32	186·6	8·01	41·5	6·08	292·7
Winter	49·57	209·3	44·82	180·6	14·44	23·7	5·61	248·1
Equinox	21·62	172·1	53·77	193·0	8·55	45·1	7·89	310·4
Summer	9·50	112·8	46·99	184·9	4·26	107·2	6·89	305·6

Table VI. does the same for the period 1905 to 1912. The inequalities are supposed to be represented by the formula

$$c_1 \sin(t+\alpha_1)+c_2 \sin(2t+\alpha_2)+\ldots,$$

t being counted from *local* midnight, and 15° being taken as the equivalent of one hour. The inequalities in Tables II. and III. referred to G.M.T., and the phase angles were first calculated in terms of G.M.T., and then corrected to local mean time by

TABLE VI.—Diurnal Inequality 1905 to 1912. Amplitudes and Phase Angles.

Month.	c_1.	α_1.	c_2.	α_2.	c_3.	α_3.	c_4.	α_4.
		°		°		°		°
January	71·06	211·6	50·99	177·3	15·94	30	7·55	271
February	46·08	200·1	60·51	173·2	18·16	0	5·62	248
March	34·88	183·0	65·09	180·5	12·14	50	8·10	294
April	36·18	157·4	61·89	183·4	5·21	91	13·05	309
May	15·64	154·5	50·59	183·6	0·97	92	8·28	319
June	6·20	148·7	38·31	184·8	4·38	72	6·06	313
July	9·77	196·7	50·29	182·4	7·86	99	8·96	279
August	2·44	188·1	54·91	188·2	4·89	131	10·07	319
September	38·01	196·9	47·63	195·3	5·02	34	7·18	324
October	35·22	229·9	41·30	212·2	16·58	39	1·60	294
November	50·22	216·0	41·67	189·0	15·77	44	7·70	258
December	58·54	216·5	36·51	193·0	12·94	31	7·56	236

adding +19′ to α_1, +38′ to α_2, and so on. 19′ represents the equivalent of the difference between Kew and Greenwich solar time, viz., about $1\frac{1}{4}$ minutes.

In the corresponding Table in E_1 (Table V., p. 311) the phase angles, it should be noticed, refer to G.M.T.

The monthly values of c_1 and c_2 from Table V. are expressed in Table VII., first in terms of their own arithmetic means, and secondly in terms of the corresponding mean monthly potential gradient. The former results are shown graphically in the two lowest curves of fig. 4. The last column of Table VII. expresses c_1 as a fraction of c_2.

TABLE VII.—Diurnal Inequality. Amplitude Ratios.

Month or season.	Ratios to their own arithmetic means.		Ratios to mean monthly values of potential gradient.		Mutual ratio.
	c_1.	c_2.	c_1.	c_2.	c_1/c_2.
January	2·03	0·94	0·141	0·113	1·25
February	1·56	1·25	0·112	0·155	0·72
March	1·02	1·32	0·085	0·188	0·45
April	0·81	1·22	0·081	0·209	0·39
May	0·73	0·96	0·087	0·197	0·44
June	0·34	0·83	0·047	0·196	0·24
July	0·10	0·98	0·014	0·230	0·06
August	0·22	1·07	0·028	0·236	0·12
September	0·95	0·92	0·110	0·184	0·60
October	0·84	0·99	0·081	0·165	0·49
November	1·37	0·75	0·101	0·095	1·07
December	2·02	0·77	0·141	0·092	1·52
Year	—	—	0·076	0·159	0·48
Winter	—	—	0·124	0·112	1·11
Equinox	—	—	0·074	0·183	0·40
Summer	—	—	0·043	0·215	0·20

When the phase angle in a Fourier wave varies much throughout the year, contributions to a diurnal inequality from different months to some extent neutralise one another. In such a case the amplitude derived from the annual or seasonal diurnal inequality may give a less accurate idea of the average size of the Fourier wave than is supplied by the arithmetic mean of the amplitudes derived from the inequalities of the individual months. For this reason arithmetic means of the c-coefficients are given in Table V., in addition to the values derived from the mean diurnal inequality for the year.

The phenomena presented by the 24-hour wave in Tables V. and VI. favour the division adopted of the year into three seasons. In the four winter months c_1 is conspicuously large and the variations in α_1 small. In the four summer months, on the other hand, c_1 is conspicuously small and α_1 is distinctly less than in winter, *i.e.*, the hours of maximum and minimum are later. The equinoctial months represent a transition from summer to winter.

In the 12-hour wave there is much less seasonal variation either in amplitude or phase angle. The amplitude of the 8-hour wave is fully as suggestive of the more usual division of the year into a summer half, April to September, and a winter half; but the phase angles favour the division adopted into three seasons. In the 6-hour wave the four winter months resemble one another in the smallness of the phase angles, but there is no marked difference in this respect between equinox and summer. There is obviously a good deal that is "accidental" in the amplitudes and phase angles obtained for individual months. Little significance, for instance, can be attached to the smallness of the July value of c_1 in Table V., or of the August value of c_1 in Table VI.

§ 12. There are marked differences between the results for c_1 and α_1 in Tables V. and VI., but as these arise from differences between the earlier and later of the 15 years they are best studied by comparing data for the periods 1898 to 1904 and 1905 to 1912. This comparison is made in Table VIII. It is confined to arithmetic means of the amplitudes, and to seasonal values of the amplitudes and phase angles, which suffice to bring out the main features. The data for 1898 to 1904 have been derived from E_1 by multiplying the amplitudes by 1·91 and altering the phase angles from G.M.T. to local mean time. If any hesitation is felt in accepting the multiplier 1·91, it may be pointed out that it brings the mean values of the potential gradient for the two periods into agreement. Practically identical conclusions would have been reached if we had expressed the Fourier amplitudes not in absolute measure, but in terms of the corresponding mean potential gradients. The multiplier does not affect the phase angles.

The outstanding feature in Table VIII. is the smallness of the differences between the results for the 12-hour wave from the two epochs. In the case, however, of the 24-hour wave the differences are obvious. The average amplitude is considerably larger in the second period than in the first, though summer shows the opposite phenomenon. In all the seasons, especially summer and equinox, the phase angle is decidedly larger in the second period, but the seasonal variation in the phase angle is markedly less in that period. The data from 1862 to 1864 discussed by EVERETT made the 24-hour wave *much* larger relative to the 12-hour wave than the 1898 to 1904 data did. Thus the 1905 to 1912 data show no progressive change, but rather a reversion.

TABLE VIII.—Comparison of Periods 1898 to 1904 and 1905 to 1912.

Period.		c_1.	α_1.	c_2.	α_2.	c_3.	α_3.	c_4.	α_4.
			°		°		°		°
Arithmetic means	1898–1904	27·57	—	48·33	—	9·20	—	6·26	—
	1905–1912	33·69	—	49·97	—	9·99	—	7·64	—
Year	1898–1904	16·04	165·4	47·27	187·5	7·30	40·4	5·35	293·9
	1905–1912	31·30	201·3	49·31	185·9	8·61	42·6	6·72	291·5
Winter	1898–1904	42·21	205·9	42·46	179·5	13·77	22·1	4·22	238·1
	1905–1912	56·14	211·5	46·96	181·6	15·04	25·2	6·92	253·5
Equinox	1898–1904	16·77	125·0	55·07	195·7	7·60	41·4	8·44	311·8
	1905–1912	32·48	191·5	52·84	190·6	9·30	48·2	7·35	307·8
Summer	1898–1904	16·85	86·4	45·34	185·0	4·37	114·6	5·69	302·9
	1905–1912	7·98	167·6	48·49	184·8	4·21	100·8	7·97	307·3

In E_1, while recognising a considerable "accidental" element in individual monthly values of amplitude and phase angle in the 8-hour and 6-hour waves, I concluded that the general features were genuine. The results from the two periods in Table VIII. show a closer resemblance than I had ventured to hope. The accordance in the phase angles is truly remarkable, considering that the time equivalent of 1° is only $1\frac{1}{3}$ minutes for α_3 and 1 minute for α_4. The 8-hour like the 24-hour term shows a well-marked seasonal variation, both in amplitude and phase angle; but unlike the 24-hour term it has the phase angle largest in summer. The 6-hour term agrees with the 12-hour term in having the phase angle least in winter, and in showing comparatively small seasonal variation of amplitude.

§ 13. To ascertain exactly how the difference between the results for the 24-hour wave from the two periods came in, I calculated the 24- and 12-hour Fourier coefficients for each individual year from 1898 to 1912. The results appear in Table IX. Fewer figures are retained than in the previous tables because the diurnal inequalities for individual years had been calculated only to 0·1 volt. The 24-hour term data fluctuate much more from year to year than do the 12-hour term data. The 1911 value of c_1 is fully five times that of 1901, while the extreme values of α_1 differ by 126°, representing $8\frac{2}{5}$ hours of time. It is the earliest four years that are mainly responsible for the smallness of α_1 for the epoch 1898 to 1904. There is, however, nothing remarkable about c_2 or α_2 in these years, and so no reason to suspect the genuineness of the results. The range in α_2 during the 15 years is only 14°, representing 28 minutes in time.

It is obvious, so far at least as Kew is concerned, that conclusions as to the relative importance of the 24- and 12-hour waves, or as to the value of the phase angle in the former wave, derived from only one or two years data, may depart considerably from average normal conditions.

TABLE IX.—FOURIER Coefficients from Individual Years.

Year.	c_1.	α_1.	c_2.	α_2.
		°		°
1898	21·6	109	54·3	189
1899	23·2	130	58·6	192
1900	15·8	156	49·7	186
1901	7·6	90	43·2	190
1902	25·9	197	36·9	180
1903	21·8	185	50·1	189
1904	30·7	212	37·8	182
1905	33·5	207	53·2	183
1906	21·6	186	55·5	189
1907	29·1	199	44·3	188
1908	24·9	181	45·6	184
1909	35·1	195	46·6	181
1910	29·5	200	48·6	186
1911	42·3	216	49·3	182
1912	37·6	211	51·6	194

§ 14. The annual variation of an element may be represented by the formula

$$M + P_1 \sin(t+\theta_1) + P_2 \sin(2t+\theta_2) + \dots ,$$

where M is the mean of the 12 monthly means, while P_1, P_2 are the amplitudes, and θ_1, θ_2 the phase angles of the annual and semi-annual terms. The time t is here measured from the beginning of the year, one month being taken as equivalent to 30°. In the calculations calendar months were treated as if of equal length, but the errors thus introduced are trifling.

Table X. contains the results obtained from the 15-year period. They are comparable with the corresponding data in E_1 (Table VI.) when the amplitudes in the latter are multiplied by 1·91. The results for the mean daily value are in very fair accordance with those in E_1, both as regards amplitude and phase angle. The variation departs but little from that given by a pure sine-wave of 12-month period, having its maximum early in January.

In the case of the diurnal inequality range the 15-year period gives nearly the same value of P_1/M as the 7-year period, but a decidedly larger value of P_2/M. In the case of c_1, P_1/M is greater, and P_2/M is less for the 15- than for the 7-year period.

In the case of c_2 the reverse holds. The phase angles from the two periods show a closer agreement than the amplitudes. The most natural inference is that even 15 years is too short a period to give a wholly normal annual variation for c_1 and c_2. This conclusion is supported by fig. 4. The c_1 and c_2 curves shown there, while much smoother than the corresponding curves in E_1, present some features which suggest abnormality, notably as regards the points representing the September values.

TABLE X.—Annual Variation. Amplitudes and Phase Angles.

Element.	P_1.	θ_1.	P_2.	θ_2.	P_1/M.	P_2/M.	P_2/P_1.
		°		°			
Mean value for the day . .	108·4	80·8	9·2	89·9	0·36	0·03	0·08$_5$
Diurnal inequality range .	31·6	56·8	14·1	334·6	0·24	0·11	0·45
,, ,, c_1 .	23·4	83·0	3·8	110·4	0·82	0·13	0·16
,, ,, c_2 .	7·2	346·2	9·6	307·8	0·15	0·20	1·35

§ 15. In a recent interesting paper* dealing with atmospheric electricity data obtained at Edinburgh during 1912, Messrs. CARSE and SHEARER compare a good many of their potential gradient results with those given for Kew in E_1. Apart altogether from the question of the absolute value of the potentials given in E_1, some doubt may be felt as to how far the results serve to compare the two stations. Messrs. CARSE and SHEARER employed all the days whose trace was uninterrupted and free from insulation troubles. As these numbered 302, in a single year, a considerable proportion presumably were days when rain fell or negative potential occurred. I am unable to say definitely how results derived from 302 days in one year at Kew would compare with the results from the 120 selected rainless days. But a recent paper by Mr. GORDON DOBSON,† which amongst other matters compares results from different species of days during 1911 at Eskdalemuir, throws some light on the question. Mr. DOBSON gives three series of diurnal variation results, the first derived from what are known as 0_a days—101 in number—whose characteristics fairly correspond with those of quiet days at Kew; the second from all ordinary days, *i.e.*, days when the record was complete and conspicuously irregular variations were absent; the third from all hourly readings irrespective of whether the day's record was complete or not.

The ordinary days—which included the 0_a days—numbered only 155, so that the difference between them and the 0_a days was very probably less than the difference one would find between all complete days at Kew and the selected quiet days. The mean potential gradients for the year derived from the three series of Eskdalemuir data were respectively 234, 219 and 185. For the four midwinter months, when 0_a

* 'Proceedings Royal Society of Edinburgh,' vol. 33, 1913, p. 317.

† 'Meteorological Office Geophysical Memoirs,' No. 7, London, 1914.

and ordinary days were relatively few, the corresponding means were 294, 278 and 212. Thus at Eskdalemuir—and I think the same would prove true at Kew—the mean potential gradient tends to fall when we include days of negative potential and of large irregular oscillations. It is thus very probable that the difference between the mean values of potential gradient at Kew and Edinburgh in 1912—respectively 300 and 167—is due in some measure to the difference in the type of days utilised.

Diurnal variation data were obtained by Mr. DOBSON from the three species of days for individual months, and for the three seasons—winter, equinox, and summer, and the results are shown graphically in his paper.

The curves for individual months and for the winter season are very irregular, but those for equinox and summer are less so. The 0_a equinoctial curve has larger ordinates than the ordinary day curve at every hour of the 24, and except for an hour or two the same is true of the summer curves. The difference in the type of the diurnal variations derived from the two series of days at these seasons does not seem to be large, but confirmation would be desirable from a period of years.

Edinburgh has a much dryer climate than Eskdalemuir, and the difference between all and rainless days there may well be less. In any case, in view of the comparison instituted by Messrs. CARSE and SHEARER with the older Kew data, it seemed worth while to compare their results with those given by the selected days of the same year at Kew, investigating at the same time how far that year was fairly representative of normal conditions.

The mean potential gradient at Kew in 1912 differed by only 4 volts from the mean for the 15 years and so was unquestionably normal.

Expressing monthly means at Edinburgh and Kew in terms of their respective mean annual values, we obtain the annual variations recorded in Table XI.

TABLE XI.—Mean Monthly Potentials as Fractions of Mean for Year.

Month	January.	February.	March.	April.	May.	June.	July.	August.	September.	October.	November.	December.
Edinburgh, 1912	1·53	1·10	0·74	1·09	1·09	0·98	0·90	0·86	0·86	0·76	1·16	0·95
Kew, 1912 . . .	1·74	1·12	0·76	0·90	0·68	0·56	0·86	0·77	1·11	1·33	1·05	1·11
Kew, 15 years .	1·35	1·30	1·13	0·94	0·78	0·68	0·69	0·73	0·81	0·97	1·27	1·35

If, following Messrs. CARSE and SHEARER, we divide the year into a 6-months summer, April to September, and a 6-months winter, we find for the ratio of the mean winter potential to the mean summer potential 1·08 : 1 at Edinburgh and 1·46 : 1 at Kew in 1912, as compared with 1·59 : 1 at Kew in the 15-year average.

The natural inference is that a comparison of winter and summer based on 1912 would not be very wide of the mark, and consequently that the difference between these two seasons is normally less at Edinburgh than at Kew.

Some individual months, however, of 1912, at Kew, diverged far from the normal. January, for instance, had an abnormally high potential, while March and December had abnormally low potentials, and the corresponding figures for Edinburgh are at least suggestive of a like phenomenon there. On the other hand, while the Kew October potential in 1912 is abnormally high, Edinburgh presents apparently exactly the opposite phenomenon.

Coming to the Fourier analysis of the diurnal inequality, we have the following results in the case of the mean diurnal inequality for the year, employing c_0 (equivalent to Messrs. CARSE and SHEARER'S α_0) to denote the mean daily value.

TABLE XII.—Comparison of Edinburgh and Kew.

	c_1/c_0.	c_2/c_0.	c_1/c_2.	α_1.	α_2.
				°	°
Edinburgh, 1912 . . .	0·142	0·102	1·39	233·1	183·0
Kew, 1912	0·125	0·172	0·73	211	194
Kew, 15 years	0·076	0·159	0·48	190·4	186·6

As Table IX. shows, the value of c_2 at Kew in 1912 was fairly normal, though a little above the average. But the 1912 value of α_2 was the largest of the 15-years, while the 1912 values of c_1 and α_1 were exceeded, the former only once, and the latter only twice.

Thus, so far at least as Kew is concerned, 1912 was hardly a year which one would have selected as representative of average conditions. Unless, however, there is a remarkable difference between all and quiet day results, there can be but little doubt that the 24-hour Fourier-wave is of greater relative importance at Edinburgh than at Kew.

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